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ON NON-HOMOGENEOUS TERNARY CUBIC DIOPHANTINE EQUATION

$$w^2 - z^2 + 2wx - 2zx = x^3$$

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***Abstract*— The non-homogeneous ternary cubic Diophantine equation $w^2 - z^2 + 2wx - 2zx = x^3$ is analyzed for its patterns of non-zero distinct integral solutions. A few relations between the solutions and special number patterns are presented.**

***Keywords*— Ternary cubic Non- Homogeneous cubic, Integral solutions.**

Notations:

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^r = \frac{n(n+1)(n(r-2)-(r-5))}{6}$$

$$CP_k^4 = \frac{4k^3 + 2k}{6}, \qquad CP_k^8 = \frac{8k^3 - 2k}{6}$$

I. INTRODUCTION

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-15] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $w^2 - z^2 + 2wx - 2zx = x^3$ representing non-homogeneous cubic with three unknowns for determining its infinitely many non-zero integral points. A few relations between the solutions and special number patterns are presented.

II. METHOD OF ANALYSIS

The given non-homogeneous ternary cubic diophantine equation is

$$w^2 - z^2 + 2wx - 2zx = x^3 \quad (1)$$

On completing the squares,(1) is written as

$$P^2 - Q^2 = x^3 \quad (2)$$

Where

$$P = w + x, Q = z + x \quad (3)$$

Write (2) as the system of double equations as below:

$$P + Q = x^3,$$

$$P - Q = 1$$

Solving the above system of equations, one obtains

$$x = 2k + 1 \quad (4)$$

and

$$P = 4k^3 + 6k^2 + 3k + 1, Q = 4k^3 + 6k^2 + 3k \quad (5)$$

In view of (3),we have

$$w = 4k^3 + 6k^2 + k, z = 4k^3 + 6k^2 + k - 1 \quad (6)$$

Thus,(4) and (6) represent the integer solutions to (1).
Relations between the solutions and special number patterns:
I. Each of the following expressions is a square multiple of 2

(i). $w - x * t_{3,2k}$

(ii). $z + 1 - x * t_{3,2k}$

(iii). $w + t_{3,2k} - 3 * P^3_{2k}$

(II). $w = -12t_{3,k} + 24 * P^3_k - k$

(III). $w = -t_{14,k} + 24 * P^3_k - 12k$

(IV). $w - x = t_{6,k} + 8 * P^5_k - 1$

(V).Each of the following is a square multiple of 6

(i). $w - 6CP^4_k + k$

(ii). $w + z + x - 6CP^8_k + 12t_{3,k}$

For simplicity and brevity, the other choices of solutions to (1) are exhibited below:

Choice 1:

$$x = k, z = t_{3,k-1} - k, w = t_{3,k} - k$$

Choice II:

$$x = 2na, z = 2n^3a^2 - (2n+1)a, w = 2n^3a^2 - (2n-1)a$$

Choice III

$$x = 2na, z = a^2 - 2(n^3 + n)a, w = a^2 + 2(n^3 - n)a$$

Choice 1V:

$$x = 2na, z = 2n^3a^3 - (2na+1), w = 2n^3a^3 - (2na-1)$$

Choice V:

$$x = 2na, z = a^3 - 2n(n^2 + a), w = a^3 + 2n(n^2 - a)$$

III.CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of ternary cubic diophantine equations.

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